1.

$$\int_{-1}^{2} 2x^3\,dx = \left[\frac{x^4}{4}\right]_{-1}^{2} = \frac{16}{4} - \left(-\frac{1}{4}\right) = 4 + \frac{1}{4} = \frac{17}{4}$$

2.

$$\int_0^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_0^{\frac{\pi}{2}} = 1 - 0 = 1$$

3.

$$\int_a^b \frac{dx}{x} = [\ln|x|]_a^b = \ln|b| - \ln|a| = \ln\left|\frac{b}{a}\right|$$

4.

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = [\arcsin x]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$$

5.

$$\int_0^{\ln 2} xe^{-x}\,dx = \left|\begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array}\right| = [-xe^{-x}]_0^{\ln 2} - \int_0^{\ln 2} \left(-e^{-x}\right)\,dx$$

$$= [-xe^{-x}]_0^{\ln 2} + \left[-e^{-x}\right]_0^{\ln 2} = -\frac{1}{2}\ln 2 - 0 - \frac{1}{2} + 1 = \frac{1}{2}(1 - \ln 2)$$

6.

$$\int_0^{\pi} x\sin x\,dx = \left|\begin{array}{ll} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array}\right|$$

$$= [-\cos x\cdot x]_0^{\pi} - \int_0^{\pi} -\cos x\,dx = [-\cos x\cdot x]_0^{\pi} + [\sin x]_0^{\pi} = \pi + 0 = \pi$$

7.

$$\int_0^1 x(1-x^2)^5 = \left|\begin{array}{lcll} 2-x^2 & = & t & x = 1 \Rightarrow t = 1 \\ -2x\,dx & = & dt & x = 0 \Rightarrow t = 2 \end{array}\right| = -\frac{1}{2}\int_2^1 t^5\,dt$$

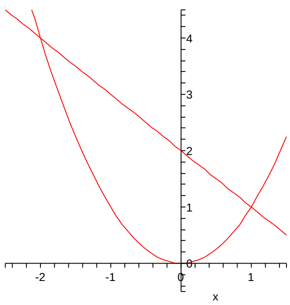
$$= \frac{1}{2}\int_1^2 t^5\,dt = \frac{1}{2}\left[\frac{t^6}{6}\right]_1^2 = \frac{64}{12} - \frac{1}{12} = \frac{63}{12}$$

8.

$$\int_1^{e^8} \frac{dx}{x\sqrt{\ln x + 1}} = \left|\begin{array}{lcll} \ln x + 1 & = & t & x = e^8 \Rightarrow t = 8+1 \\ \frac{1}{x}\,dx & = & dt & x = 1 \Rightarrow t = 0+1 \end{array}\right| = \int_1^9 \frac{dt}{\sqrt{t}} = \left[2\sqrt{t}\right]_1^9 = 6 - 2 = 4$$

9. Vypočtěte obsah plochy ohraničené křivkami

$$y = 2 - x, y = x^2$$



$$2 - x = x^2 \rightarrow$$

$$0 = x^2 + x - 2 \rightarrow$$

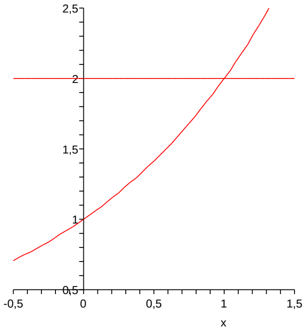
$$0 = (x+2)(x-1)$$

$$\int_{-2}^{1} (2 - x - x^2)\,dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$$

$$= 2 - \frac{1}{2} - \frac{1}{3} - \left(-4 - \frac{4}{2} - \frac{-8}{3}\right) = 5 - \frac{1}{2} = \frac{9}{2}$$

10.

$$y = 2^x, y = 2, x = 0$$



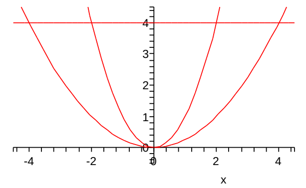
$$2^x = 2 \Rightarrow x = 1$$

$$\int_0^1 (2 - 2^x)\, dx = \left[2x - \frac{2^x}{\ln 2}\right]_0^1$$

$$= 2 - \frac{2}{\ln 2} - \left(0 - \frac{1}{\ln 2}\right) = 2 - \frac{1}{\ln 2}$$

11.

$$y = x^2, y = \frac{1}{4}x^2, y = 4$$



$$x^2 = \frac{1}{4}x^2 \Rightarrow x = 0$$

$$4 = x^2 \Rightarrow x = \pm 2$$

$$4 = \frac{1}{4}x^2 \Rightarrow x = \pm 4$$

$$S = 2S_1$$

$$S_1 = \int_0^2 \left(x^2 - \frac{x^2}{4}\right) dx + \int_2^4 \left(4 - \frac{x^2}{4}\right) dx = \left[\frac{x^3}{3} - \frac{x^3}{12}\right]_0^2 + \left[4x - \frac{x^3}{12}\right]_2^4 = \frac{8}{3} - \frac{8}{12} + 16 - \frac{16}{3} - 8 + \frac{8}{12} = \frac{16}{3}$$

$$S = \frac{32}{3}$$

12. Vypočtěte objem komolého kužele s poloměrem podstav r_1, r_2 a výškou v.
Lichoběžník $[0, r_2], [v, r_1], [0, 0], [v, 0]$ necháme rotovat kolem osy x.

$$y = kx + q, \quad k = \frac{r_1 - r_2}{v}, \quad q = r_2$$

$$y = \frac{r_1 - r_2}{v}x + r_2$$

$$V = \pi \int_a^b f^2(x)\, dx = \pi \int_0^v \left(\frac{r_1 - r_2}{v}x + r_2\right)^2 dx = \pi \int_0^v \left(\frac{(r_1 - r_2)^2}{v^2}x^2 + 2\frac{(r_1 - r_2)r_2}{v}x + r_2^2\right) dx$$

$$= \pi \left[\frac{(r_1 - r_2)^2}{3v^2}x^3 + \frac{(r_1 - r_2)r_2}{v}x^2 + r_2^2 x\right]_0^v = \frac{1}{3}\pi v(r_1^2 + r_1 r_2 + r_2^2)$$

13. Vypočtěte délku obkouku křivky

$$y = \ln x, x_1 = \sqrt{3}, x_2 = \sqrt{8}.$$

$$l = \int_{x_1}^{x_2} \sqrt{1 + (f')^2}\, dx = \int_{\sqrt{3}}^{\sqrt{8}} \sqrt{1 + \frac{1}{x^2}}\, dx = \int_{\sqrt{3}}^{\sqrt{8}} \frac{\sqrt{1 + x^2}x}{x^2}\, dx$$

$$= \left| \begin{array}{lll} 1+x^2 & = & t^2 \qquad x=\sqrt{8} \Rightarrow t=3 \\ 2x\,dx & = & 2t\,dt \quad x=\sqrt{3} \Rightarrow t=2 \end{array} \right| = \int_2^3 \frac{t^2}{t^2-1}\,dt = \int_2^3 \left(1+\frac{1}{t^2-1}\right)\,dt$$

$$= \int_2^3 \,dt + \frac{1}{2}\int_2^3 \left(\frac{1}{t-1}-\frac{1}{t+1}\right)\,dt == [t]_2^3 + \frac{1}{2}\left[\ln(t-1)-\ln(t+1)\right]_2^3 = 1+\frac{1}{2}\ln\left(\frac{3}{2}\right)$$