

Vypočítejte první a druhé parciální derivace funkcí:

1.  $f(x, y) = 3(x)^2 - 5(y)^3 + 1$

$$\frac{\partial f(x, y)}{\partial x} = 6x \quad \frac{\partial f(x, y)}{\partial y} = -15(y)^2$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} = 6 \quad \frac{\partial^2 f(x, y)}{\partial y^2} = -30y \quad \frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = 0$$

2.  $f(x, y) = 4\sqrt[3]{x^5} - \ln y^2 = 4x^{\frac{5}{3}} - 2 \ln y$

$$\frac{\partial f(x, y)}{\partial x} = \frac{20}{3}x^{\frac{2}{3}} \quad \frac{\partial f(x, y)}{\partial y} = -\frac{2}{y}$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} = \frac{40}{9}x^{-\frac{1}{3}} \quad \frac{\partial^2 f(x, y)}{\partial y^2} = \frac{2}{y^2} \quad \frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = 0$$

3.  $f(x, y) = 5x^3y^2 - x^2y^3$

$$\frac{\partial f(x, y)}{\partial x} = 15x^2y^2 - 2xy^3 \quad \frac{\partial f(x, y)}{\partial y} = 10x^3y - 3x^2y^2$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} = 30xy^2 - 2y^3 \quad \frac{\partial^2 f(x, y)}{\partial y^2} = 10x^3 - 6x^2y$$

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = 30x^2y - 6xy^2$$

4.  $f(x, y) = x^y$

$$\frac{\partial f(x, y)}{\partial x} = yx^{y-1} \quad \frac{\partial f(x, y)}{\partial y} = x^y \ln x$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} = y(y-1)x^{y-2} \quad \frac{\partial^2 f(x, y)}{\partial y^2} = \ln x x^y \ln x = x^y \ln^2 x$$

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = yx^{y-1} \ln x + x^{y-1}$$

5.  $f(x, y) = x \sin(x + y)$

$$\frac{\partial f(x, y)}{\partial x} = \sin(x + y) + x \cos(x + y) \quad \frac{\partial f(x, y)}{\partial y} = x \cos(x + y)$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} = \cos(x + y) + \cos(x + y) + x \sin(x + y) = 2 \cos(x + y) + x \sin(x + y)$$

$$\frac{\partial^2 f(x, y)}{\partial y^2} = -x \sin(x + y)$$

$$\frac{\partial^2 f(x, y)}{\partial x \partial y} = \frac{\partial^2 f(x, y)}{\partial y \partial x} = \cos(x + y) - x \sin(x + y)$$

Vypočítejte první parciální derivace funkcí:

6.  $f(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$

$$\frac{\partial f(x, y)}{\partial x} = \frac{3x^2(x^2 + y^2) - (x^3 + y^3)2x}{(x^2 + y^2)^2} = \frac{3x^4 + 3x^2y^2 - 2x^4 - 2xy^3}{(x^2 + y^2)^2} = \frac{x^4 + 3x^2y^2 - 2xy^3}{(x^2 + y^2)^2}$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{3y^2(x^2 + y^2) - (x^3 + y^3)2y}{(x^2 + y^2)^2} = \frac{3x^2y^2 + 3y^4 - 2x^3y - 2y^4}{(x^2 + y^2)^2} = \frac{y^4 + 3x^2y^2 - 2x^3y}{(x^2 + y^2)^2}$$

7.  $f(x, y) = (\sin x)^{\cos y}$

$$\frac{\partial f(x, y)}{\partial x} = \cos y (\sin x)^{\cos y - 1} \cos x \quad \frac{\partial f(x, y)}{\partial y} = -\sin y (\sin x)^{\cos y} \ln(\sin x)$$

8.  $f(x, y) = \tan\left(\frac{x^2}{y}\right)$

$$\frac{\partial f(x, y)}{\partial x} = \frac{1}{\cos^2\left(\frac{x^2}{y}\right)} \frac{2x}{y} \quad \frac{\partial f(x, y)}{\partial y} = \frac{1}{\cos^2\left(\frac{x^2}{y}\right)} \frac{-x^2}{y^2}$$

9.  $f(x, y) = \left(\frac{1}{3}\right)^{\frac{y}{x}}$

$$\frac{\partial f(x, y)}{\partial x} = \left(\frac{1}{3}\right)^{\frac{y}{x}} \ln 3 \frac{y}{x^2} \quad \frac{\partial f(x, y)}{\partial y} = -\left(\frac{1}{3}\right)^{\frac{y}{x}} \ln 3 \frac{1}{x}$$

10.  $f(x, y) = xe^{\frac{y}{x}}$

$$\frac{\partial f(x, y)}{\partial x} = e^{\frac{y}{x}} - xe^{\frac{y}{x}} \frac{y}{x^2} \quad \frac{\partial f(x, y)}{\partial y} = xe^{\frac{y}{x}} \frac{1}{x} = e^{\frac{y}{x}}$$

11.  $f(x, y) = \ln \frac{x - y}{x + y}$

$$\frac{\partial f(x, y)}{\partial x} = \frac{1}{\frac{x-y}{x+y}} \frac{(x+y) - (x-y)}{(x+y)^2} = \frac{2y}{(x^2 - y^2)}$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{1}{\frac{x-y}{x+y}} \frac{-1(x+y) - (x-y)}{(x+y)^2} = \frac{-2x}{(x^2 - y^2)}$$

12.  $f(x, y) = \ln(x + \sqrt{x^2 + y^2})$

$$\frac{\partial f(x, y)}{\partial x} = \frac{1}{x + \sqrt{x^2 + y^2}} \left(1 + \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2}}\right) = \frac{1}{x + \sqrt{x^2 + y^2}} \left(\frac{\sqrt{x^2 + y^2} + x}{\sqrt{x^2 + y^2}}\right) = \frac{1}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{1}{x + \sqrt{x^2 + y^2}} \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2}} = \frac{y}{x\sqrt{x^2 + y^2} + x^2 + y^2}$$

13.  $f(x, y) = \arcsin \frac{x}{\sqrt{x^2 + y^2}}$

$$\begin{aligned}\frac{\partial f(x, y)}{\partial x} &= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + y^2}}\right)^2}} \frac{\sqrt{x^2 + y^2} - x \frac{1}{2} \frac{1}{\sqrt{x^2 + y^2}} 2x}{x^2 + y^2} = \\ &= \frac{1}{\sqrt{\frac{x^2 + y^2 - x^2}{x^2 + y^2}}} \frac{x^2 + y^2 - x^2}{(x^2 + y^2) \sqrt{x^2 + y^2}} = \frac{y}{(x^2 + y^2) \sqrt{x^2 + y^2}} \\ \frac{\partial f(x, y)}{\partial y} &= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + y^2}}\right)^2}} x \frac{-1}{2} \frac{1}{\sqrt{(x^2 + y^2)^3}} 2y = \\ &= \frac{1}{\sqrt{\frac{x^2 + y^2 - x^2}{x^2 + y^2}}} \frac{-xy}{(x^2 + y^2) \sqrt{x^2 + y^2}} = \frac{-x}{x^2 + y^2}\end{aligned}$$

14.  $f(x, y) = \arctan \frac{x - y}{x + y}$

$$\begin{aligned}\frac{\partial f(x, y)}{\partial x} &= \frac{1}{1 + \left(\frac{x-y}{x+y}\right)^2} \frac{(x+y) - (x-y)}{(x+y)^2} = \\ &= \frac{(x+y)^2}{x^2 + 2yx + y^2 + x^2 - 2yx + y^2} \frac{x+y-x+y}{(x+y)^2} = \frac{y}{x^2 + y^2} \\ \frac{\partial f(x, y)}{\partial y} &= \frac{1}{1 + \left(\frac{x-y}{x+y}\right)^2} \frac{-1(x+y) - (x-y)}{(x+y)^2} = \\ &= \frac{(x+y)^2}{x^2 + 2yx + y^2 + x^2 - 2yx + y^2} \frac{-x-y-x+y}{(x+y)^2} = \frac{-x}{x^2 + y^2}\end{aligned}$$

15.  $f(x, y) = (x + y)^y$

Využijeme vzorce:  $A^B = e^{B \ln A}$

$$f(x, y) = e^{y \ln(x+y)}$$

$$\frac{\partial f(x, y)}{\partial x} = e^{y \ln(x+y)} y \frac{1}{x+y} \quad \frac{\partial f(x, y)}{\partial y} = e^{y \ln(x+y)} \left[ \ln(x+y) + y \frac{1}{x+y} \right]$$

16.  $f(x, y) = (x)^{x^y}$

Využijeme vzorce:  $A^B = e^{B \ln A}$

$$f(x, y) = e^{x^y \ln x}$$

$$\frac{\partial f(x, y)}{\partial x} = e^{x^y \ln x} \left( y x^{y-1} \ln x + x^y \frac{1}{x} \right) \quad \frac{\partial f(x, y)}{\partial y} = e^{x^y \ln x} (\ln x)^2 x^y$$

17.  $f(x, y) = (x)^{x^y}$

Vyulijeme vzorce:  $A^B = e^{B \ln A}$

$$f(x, y) = e^{x^y \ln x}$$

$$\frac{\partial f(x, y)}{\partial x} = e^{x^y \ln x} \left( yx^{y-1} \ln x + x^y \frac{1}{x} \right) \quad \frac{\partial f(x, y)}{\partial y} = e^{x^y \ln x} (\ln x)^2 x^y$$

18.  $f(x, y) = xy \cdot e^{\sin(\pi xy)}$

$$\frac{\partial f(x, y)}{\partial x} = y \cdot e^{\sin(\pi xy)} [1 + \pi xy \cos(\pi xy)]$$

$$\frac{\partial f(x, y)}{\partial y} = x \cdot e^{\sin(\pi xy)} [1 + \pi xy \cos(\pi xy)]$$

19.  $f(x, y) = \ln \frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}$

$$\frac{\partial f(x, y)}{\partial x} = \frac{1}{\frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}} \frac{\left( \frac{1}{2} \frac{1}{\sqrt{x^2 + y^2}} 2x - 1 \right) (\sqrt{x^2 + y^2} + x) - (\sqrt{x^2 + y^2} - x) \left( \frac{1}{2} \frac{1}{\sqrt{x^2 + y^2}} 2x + 1 \right)}{(\sqrt{x^2 + y^2} + x)^2} =$$

$$= \frac{(x - \sqrt{x^2 + y^2})(x + \sqrt{x^2 + y^2}) - (\sqrt{x^2 + y^2} - x)(\sqrt{x^2 + y^2} + x)}{(\sqrt{x^2 + y^2} + x)(\sqrt{x^2 + y^2} - x)\sqrt{x^2 + y^2}} =$$

$$= \frac{(x - \sqrt{x^2 + y^2})(x + \sqrt{x^2 + y^2}) + (x - \sqrt{x^2 + y^2})(x + \sqrt{x^2 + y^2})}{y^2 \sqrt{x^2 + y^2}} =$$

$$= \frac{[x^2 - (x^2 + y^2)] + [x^2 - (x^2 + y^2)]}{y^2 \sqrt{x^2 + y^2}} = \frac{-2}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f(x, y)}{\partial y} = \frac{\sqrt{x^2 + y^2} + x}{\sqrt{x^2 + y^2} - x} \frac{\left( \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2}} \right) [(\sqrt{x^2 + y^2} + x) - (\sqrt{x^2 + y^2} - x)]}{(\sqrt{x^2 + y^2} + x)^2} =$$

$$= \frac{2xy}{(\sqrt{x^2 + y^2} - x)(\sqrt{x^2 + y^2} + x)} = \frac{2x}{y\sqrt{x^2 + y^2}}$$

Vypočítejte parciální derivace tří proměnných:

20.  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial f(x, y, z)}{\partial x} = \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\partial f(x, y, z)}{\partial y} = \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2 + z^2}} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{\partial f(x, y, z)}{\partial z} = \frac{1}{2} \frac{2z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

21.  $f(x, y, z) = (xy)^z$

Využijeme vzorce:  $A^B = e^{B \ln A}$

$$f(x, y, z) = e^{z \ln(xy)}$$

$$\frac{\partial f(x, y, z)}{\partial x} = e^{z \ln(xy)} z \frac{1}{xy} y = e^{z \ln(xy)} \frac{z}{x} = (xy)^z \frac{z}{x}$$

$$\frac{\partial f(x, y, z)}{\partial y} = e^{z \ln(xy)} z \frac{1}{xy} x = e^{z \ln(xy)} \frac{z}{y} = (xy)^z \frac{z}{y}$$

$$\frac{\partial f(x, y, z)}{\partial z} = e^{z \ln(xy)} \ln(xy) = (xy)^z \ln(xy)$$