Vypočtěte parciální derivace 1. řádu

1. $z = x^3 - x^2y + xy^2 + 3y^2 - 6x + y - 3$
   $z_x = 3x^2 - 2xy + y^2 + 0 - 6 + 0 + 0 = 3x^2 - 2xy + y^2 - 6$
   $z_y = 0 - x^2 + x2y + 6y - 0 + 1 - 0 = -x^2 + 2xy + 6y + 1$

2. $z = x\cos(3x - 2y)$
   $z_x = 1 \cdot \cos(3x-2y) + x \cdot (-\sin(3x-2y)) \cdot 3 = \cos(3x-2y) - 3x\sin(3x-2y)$
   $z_y = x \cdot (-\sin(3x-2y)) \cdot (-2) = 2x\sin(3x-2y)$

3. $z = \frac{x}{y}$
   $z_x = \frac{1}{y}, \quad z_y = x \cdot (-1)y^{-2} = -\frac{x}{y^2}$

4. $z = \frac{\sqrt{y}+2xy}{\sqrt{x}}$
   $z_x = \frac{(0+2\sqrt{y})\sqrt{x} - (\sqrt{y}+2xy)\frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \frac{\frac{4x\sqrt{y}-\sqrt{y}-2xy}{2\sqrt{x}}}{x} = \frac{4x\sqrt{y}-\sqrt{y}-2xy}{2x\sqrt{x}}$
   $z_y = \frac{1}{\sqrt{x}}\frac{1}{2\sqrt{y}} + \frac{2x}{\sqrt{x}} = \frac{1}{2\sqrt{xy}} + 2\sqrt{x}$

5. $z = \sin\frac{x}{y} \cdot \cos\frac{y}{x}$
   $z_x = \cos\frac{x}{y} \cdot \frac{1}{y} \cdot \cos\frac{y}{x} + \sin\frac{x}{y} \cdot \left(-\sin\frac{y}{x}\right) \cdot \left(-\frac{y}{x^2}\right)$
   $z_y = \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right) \cdot \cos\frac{y}{x} + \sin\frac{x}{y} \cdot \left(-\sin\frac{y}{x}\right) \cdot \frac{1}{x}$

6. $z = \mathrm{e}^{-\frac{y}{x}} + x^y$
   $z_x = \mathrm{e}^{-\frac{y}{x}}\frac{y}{x^2} + yx^{y-1}$
   $z_y = \mathrm{e}^{-\frac{y}{x}}\left(-\frac{1}{x}\right) + x^y\ln x$

7. $z = \ln(x + x\sqrt{y})$
   $z_x = \frac{1}{x+x\sqrt{y}}(1+\sqrt{y}) = \frac{1}{x}$
   $z_y = \frac{1}{x+x\sqrt{y}}\frac{x}{2\sqrt{y}} = \frac{1}{2(\sqrt{y}+y)}$

8. $z = \operatorname{arctg}\frac{x}{y}$
   $z_x = \frac{1}{1+\frac{x^2}{y^2}}\frac{1}{y} = \frac{1}{\frac{y^2+x^2}{y^2}}\frac{1}{y} = \frac{y}{x^2+y^2}$
   $z_y = \frac{1}{1+\frac{x^2}{y^2}}\left(-\frac{x}{y^2}\right) = -\frac{1}{\frac{y^2+x^2}{y^2}}\frac{x}{y^2} = -\frac{x}{x^2+y^2}$

9. $u = \sin(x^2+y^2+z^2)$
   $u_x = \cos(x^2+y^2+z^2) \cdot 2x, \quad u_y = \cos(x^2+y^2+z^2) \cdot 2y, \quad u_z = \cos(x^2+y^2+z^2) \cdot 2z$

10. $u = \left(\frac{y}{z}\right)^x$
    $u_x = \left(\frac{y}{z}\right)^x \ln\left(\frac{y}{z}\right), \quad u_y = x\left(\frac{y}{z}\right)^{x-1}\frac{1}{z}, \quad u_z = x\left(\frac{y}{z}\right)^{x-1}\left(-\frac{y}{z^2}\right)$

Vypočtěte parciální derivace prvního řádu v bodě A

1. $z = \frac{xy}{y-x}, \quad A = [2,3]$
   $z_x = \frac{y(y-x)-xy(-1)}{(y-x)^2} = \frac{y^2}{(y-x)^2}, \quad z_x(2,3) = 9$
   $z_y = \frac{x(y-x)-xy}{(y-x)^2} = \frac{-x^2}{(y-x)^2}, \quad z_y(2,3) = -4$

2. $z = \ln(x - y\,\mathrm{e}^x), \quad A = [1,1]$
   $z_x = \frac{1}{(x-y\,\mathrm{e}^x)}(1 - y\,\mathrm{e}^x) = \frac{1-y\,\mathrm{e}^x}{x-y\,\mathrm{e}^x}, \quad z_x(1,1) = \frac{1-\mathrm{e}}{1-\mathrm{e}} = 1$
   $z_y = \frac{1}{(x-y\,\mathrm{e}^x)}(-\mathrm{e}^x) = \frac{-\mathrm{e}^x}{x-y\,\mathrm{e}^x}, \quad z_y(1,1) = \frac{-e}{1-\mathrm{e}} = \frac{\mathrm{e}}{\mathrm{e}-1}$

3. $z = \sqrt{x}\sin\frac{y}{x}, \quad A = [4,\pi]$
   $z_x = \frac{1}{2\sqrt{x}}\sin\frac{y}{x} + \sqrt{x}\cos\frac{y}{x}\left(\frac{-y}{x^2}\right), \quad z_x(4,\pi) = \frac{1}{4}\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2}\left(-\frac{\pi}{16}\right) = \frac{\sqrt{2}}{8} - \frac{\sqrt{2}}{16}\pi$
   $z_y = \sqrt{x}\cos\frac{y}{x} \cdot \frac{1}{x}, \quad z_y(4,\pi) = 2\frac{\sqrt{2}}{2}\frac{1}{4} = \frac{\sqrt{2}}{4}$

4. $u = x^{y^z}, \quad A = [1, \mathrm{e}, 1]$
$u_x = y^z \cdot x^{y^z - 1}, \quad u_x(1, \mathrm{e}, 1) = \mathrm{e}$
$u_y = x^{y^z} \cdot \ln x \cdot z y^{z-1}, \quad u_y(1, \mathrm{e}, 1) = 0$
$u_z = x^{y^z} \cdot \ln (x^y), \quad u_z(1, \mathrm{e}, 1) = 0$

Najděte parciální derivace 1. a 2. řádu funkce

1.

$$z = \frac{\cos x^2}{y}$$

$$z_x = \frac{1}{y}(-\sin x^2)2x = -\frac{2x \sin x^2}{y}, \quad z_y = -\frac{\cos x^2}{y^2}$$

$$z_{xx} = -\frac{2 \sin x^2 + 2x \cos x^2 \cdot 2x}{y}, \quad z_{yy} = 2\frac{\cos x^2}{y^3}$$

$$z_{xy} = \frac{2x \sin x^2}{y^2}, \quad z_{yx} = -\frac{(-\sin x^2) \cdot 2x}{y^2} = \frac{2x \sin x^2}{y^2} = z_{xy}$$

2.

$$z = \arcsin \frac{x+y}{xy}$$

$$z_x = \frac{1}{\sqrt{1 - \frac{(x+y)^2}{x^2y^2}}} \frac{xy - (x+y)y}{x^2y^2} = \frac{xy}{\sqrt{1-(x+y)^2}} \frac{-y^2}{x^2y^2} = \frac{-y}{x\sqrt{1-(x+y)^2}}$$

$$z_y = \frac{1}{\sqrt{1 - \frac{(x+y)^2}{x^2y^2}}} \frac{xy - (x+y)x}{x^2y^2} = \frac{xy}{\sqrt{1-(x+y)^2}} \frac{-x^2}{x^2y^2} = \frac{-x}{y\sqrt{1-(x+y)^2}}$$

$$z_{xx} = \frac{-y}{x^2(1-(x+y)^2)} \left(\sqrt{1-(x+y)^2} + \frac{x}{2\sqrt{1-(x+y)^2}}(-2(x+y)) \right)$$

$$z_{yy} = \frac{-x}{y^2(1-(x+y)^2)} \left(\sqrt{1-(x+y)^2} + \frac{y}{2\sqrt{1-(x+y)^2}}(-2(x+y)) \right)$$

$$z_{xy} = \frac{-x\sqrt{1-(x+y)^2} + y\frac{x}{2\sqrt{1-(x+y)^2}}(-2(x+y))}{x^2(1-(x+y)^2)}$$