Vypočítejte limity:

$$\lim_{(x,y)\to(1,0)} \frac{x+y+1}{x+y+3} = \frac{1}{2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = 2$$

$$\lim_{(x,y)\to(0,0)} (x+y)\cdot \sin\frac{1}{x}\cdot \sin\frac{1}{y} = 0$$

$$\lim_{(x,y)\to(1,2)} \frac{\sin(x-y+1)}{x-y+1} = 1$$

$$\lim_{(x,y)\to(1,1)} \frac{x+y}{\sqrt{x^2+y^2}} = 1$$

$$\lim_{(x,y)\to(e^2,1)} \frac{\ln x}{y} = 2$$

$$\lim_{(x,y)\to(1,0)} \frac{\ln(x+e^y)}{\sqrt{x^2+y^2}} = \ln 2$$

$$\lim_{(x,y)\to(-4,-1)} \frac{(x-y)^2-9}{x^2+y^2} = 0$$

$$\lim_{(x,y)\to(0,0)} xy^2\cos\frac{1}{xy^2} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2y^2+1}-1}{x^2+y^2} = 0$$

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)^{x^2y^2} = 1$$

$$\lim_{(x,y)\to(\infty,\infty)} \frac{x-y}{x^2-xy+y^2} = 0$$

$$\lim_{(x,y)\to(0,2)} \frac{\sin(xy)}{x} = 2$$

$$\lim_{(x,y)\to(\infty,\infty)} \frac{x^2+y^2}{x^4+y^4} = 0$$

$$\lim_{(x,y)\to(0,3)} \frac{e^{xy}-1}{x} = 3$$

Ukažte, že následující limity neexistují:

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x-2y}{3x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+x^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x+y}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{3y}{x^3+y}$$

Určete body nespojitosti funkcí:

$$f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$$

body nespojitosti: $(0,0)$

$$f(x,y) = \frac{x+y}{x^3+y^3}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x = -y\}$

$$f(x,y) = \sin\frac{1}{xy}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x = 0 \vee y = 0\}$

$$f(x,y) = \frac{1}{\sin x \cdot \sin y}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x = k\pi, y = k\pi, k \in \mathbb{Z}\}$

$$f(x,y) = \ln|1 - x^2 - y^2|$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 1\}$

$$f(x,y) = \frac{x^2+y^5+x+3}{x^4+xy^3}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x = 0 \vee x = -y\}$

$$f(x,y) = \frac{x^2+3y}{x^2-3y}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; y = \frac{x^2}{3}\}$

$$f(x,y) = \frac{1}{e^{\frac{x}{y}-1}}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; x = 0 \vee y = 0\}$

$$f(x,y) = \arccos\frac{x}{y}$$

body nespojitosti: $\{(x,y) \in \mathbb{R}^2; y = 0\}$

Zjistěte, zda je funkce f v bodě $(0,0)$ spojitá:

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2} & \text{pro } (x,y) \neq (0,0) \\ 0 & \text{pro } (x,y) = (0,0) \end{cases}$$

Ř: je spojitá

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & \text{pro } (x,y) \neq (0,0) \\ 0 & \text{pro } (x,y) = (0,0) \end{cases}$$

Ř: není spojitá