$$\int (3-x^2)^3\,dx = 27x - 9x^3 + \frac{9}{5}x^5 - \frac{1}{7}x^7 + c$$

$$\int (1+\sin x+\cos x)\,dx = x - \cos x + \sin x + c$$

$$\int (2^x+3^x)^2\,dx = \frac{1}{\ln 4}4^x + 2\frac{1}{\ln 6}6^x + \frac{1}{\ln 9}9^x + c$$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + c$$

$$\int \frac{x^2}{1+x^2}\,dx = x - \operatorname{arctg} x + c$$

$$\int \operatorname{cotg}^2 x\,dx = -\operatorname{cotg} x - x + c$$

$$\int \sqrt[3]{1-3x}\,dx = -\frac{1}{4}\sqrt[3]{(1-3x)^4} + c$$

$$\int \frac{1}{1+\cos x}\,dx = -\operatorname{cotg} x + \frac{1}{\sin x} + c$$

$$\int \frac{1}{2+3x^2}\,dx = \frac{1}{\sqrt{6}}\operatorname{arctg}\left(\sqrt{\frac{3}{2}}x\right) + c$$

$$\int \frac{1}{2-3x^2}\,dx = \frac{1}{2\sqrt{6}}\ln\left|\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right| + c$$

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + c$$

$$\int \frac{x}{4+x^4}\,dx = \frac{1}{4}\operatorname{arctg}\frac{x^2}{2} + c$$

$$\int \frac{1}{(1+x)\sqrt{x}}\,dx = 2\operatorname{arctg}\sqrt{x} + c$$

$$\int \frac{1}{x\sqrt{x^2+1}}\,dx = \ln\left|\frac{\sqrt{x^2+1}-1}{x}\right| + c$$

$$\int \frac{x^2}{(8x^3+27)^{2/3}}\,dx = \frac{1}{8}\sqrt[3]{8x^3+27} + c$$

$$\int xe^{-x^2}\,dx = -\frac{1}{2}e^{-x^2} + c$$

$$\int \frac{\sin x}{\sqrt{\cos^3 x}}\,dx = \frac{2}{\sqrt{\cos x}} + c$$

$$\int \frac{\ln^2 x}{x}\,dx = \frac{1}{3}\ln^3 x + c$$

$$\int \frac{1}{e^x+e^{-x}}\,dx = \operatorname{arctg} e^x + c$$

$$\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{\cos x-1}{\cos x+1}\right| + c$$

$$\int \frac{x^3}{x+3}\,dx = \frac{x^3}{3} - \frac{3x^2}{2} + 9x - 27\ln|x+3| + c$$

$$\int \frac{(2-x)^2}{2-x^2}\,dx = -x + 2\ln|x^2-2| + \frac{3}{\sqrt{2}}\ln\left|\frac{x+\sqrt{2}}{x-\sqrt{2}}\right| + c$$

$$\int \ln x\,dx = x(\ln x - 1) + c$$

$$\int \left(\frac{\ln x}{x}\right)^2\,dx = -\frac{1}{x}(\ln^2 x + 2\ln x + 2) + c$$

$$\int xe^{-x}\,dx = -e^{-x}(x+1) + c$$

$$\int x^2\sin 2x\,dx = -\frac{2x^2-1}{4}\cos 2x + \frac{x}{2}\sin 2x + c$$

$$\int x\ln\frac{1+x}{1-x}\,dx = x + \frac{x^2-1}{2}\ln\left|\frac{1+x}{1-x}\right| + c$$

$$\int \ln(x+\sqrt{1+x^2})\,dx = x\ln(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + c$$

$$\int \cos(\ln x)\,dx = \frac{1}{2}x(\cos\ln x + \sin\ln x) + c$$

$$\int x^5 e^{x^3}\,dx = \frac{1}{3}e^{x^3}(x^3-1) + c$$

$$\int e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(\sqrt{x}-1) + c$$

$$\int \frac{2x+3}{(x-2)(x+5)}\,dx = \ln|x-2| + \ln|x+5| + c$$

$$\int \frac{x^4}{x^4+5x^2+4}\,dx = x + \frac{1}{3}\operatorname{arctg} x - \frac{8}{3}\operatorname{arctg}\frac{x}{2} + c$$

$$\int \frac{1}{x^3+1}\,dx = \frac{\ln|x+1|}{3} - \frac{\ln|x^2-x+1|}{6} + \frac{1}{\sqrt{3}}\operatorname{arctg}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

$$\int \frac{x^2}{(x^2+2x+2)^2}\,dx = \operatorname{arctg}(x+1) + \frac{1}{x^2+2x+2} + c$$

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\ln(\sqrt{x}+1) + c$$

$$\int \frac{dx}{x(1+2\sqrt{x}+\sqrt[3]{x})} = \frac{3}{4}\ln\frac{x\sqrt[3]{x}}{(1+\sqrt[6]{x})^2(\frac{1}{2}-\frac{1}{2}\sqrt[6]{x}+\sqrt[3]{x})^3} -$$

$$-\frac{3}{2\sqrt{7}}\text{arctg}\left(\frac{4\sqrt[6]{x}-1}{\sqrt{7}}\right) + c$$

$$\int \frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}}\,dx = -\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}} + c$$

Návod: použijte substituci $t = \sqrt[3]{\frac{x+1}{x-1}}$

$$\int \cos x \cos 2x\,dx = \frac{1}{2}\sin x + \frac{1}{6}\sin 3x + c$$

$$\int \cos x \cos 2x \cos 3x\,dx = \frac{x}{4} + \frac{\sin 2x}{8} + \frac{\sin 4x}{16} + \frac{\sin 6x}{24} + c$$

$$\int \frac{\sin^3 x}{\cos^4 x}\,dx = \frac{1}{3\cos^3 x} - \frac{1}{\cos x} + c$$

$$\int \cos^5 x\,dx = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

$$\int \frac{\sin^2 x}{1+\sin^2 x}\,dx = x - \frac{1}{\sqrt{2}}\text{arctg}\,(\sqrt{2}\text{tg}\,x) + c$$

$$\int \frac{1}{2\sin x - \cos x + 5}\,dx = \frac{1}{\sqrt{5}}\text{arctg}\left(\frac{3\text{tg}\,\frac{x}{2}+1}{\sqrt{5}}\right) + c$$

$$\int \text{tg}^3 x\,dx = \frac{1}{2}\frac{1}{\cos^2 x} + \ln|\cos x| + c$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

$$\int (x^2-2x+2)e^{-x}\,dx = -e^{-x}(x^2+2) + c$$

$$\int x^2 e^{\sqrt{x}}\,dx = 2e^{\sqrt{x}}(x^2\sqrt{x} - 5x^2 + 20x\sqrt{x} - 60x + 120\sqrt{x} - 120) + c$$

$$\int \frac{1}{(1+e^x)^2}\,dx = x - \ln(e^x+1) + \frac{1}{1+e^x} + c$$

$$\int_{-1}^{8} \sqrt[3]{x}\,dx = \frac{45}{4}$$

$$\int_{0}^{\pi} \sin x\,dx = 2$$

$$\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \frac{\pi}{6}$$

$$\int_{0}^{\pi} x\sin x\,dx = \pi$$

$$\int_{0}^{\ln 2} \sqrt{e^x-1}\,dx = 2 - \frac{\pi}{2}$$

$$\int_{-1}^{1} \frac{x}{\sqrt{5-4x}}\,dx = \frac{1}{6}$$

$$\int_{0}^{1} \frac{1}{x^2+x+1}\,dx = \frac{\pi}{3\sqrt{3}}$$

$$\int_{1}^{9} x\sqrt[3]{1-x}\,dx = -\frac{468}{7} = -66\frac{6}{7}$$

$$\int_{1}^{e} (x\ln x)^2\,dx = \frac{5e^3-2}{27}$$

$$\int_{0}^{\pi} e^x\cos^2 x\,dx = \frac{e^\pi - 1}{5}$$

Určete obsah plochy omezený danými křivkami:

a) $y = x^2, x+y = 2$ $S = \frac{9}{2}$

b) $y = 2^x, y = 2, x = 0$ $S = 2 - \frac{1}{\ln 2}$

c) $y = x, y = x + \sin^2 x, 0 \le x \le \pi$ $S = \frac{1}{2}\pi$

Vypočtěte objem tělesa ohraničeného plochami, které vzniknou rotací daných křivek kolem osy x:

a) $y = 2x - x^2, y = 0$ $S = \frac{16}{15}\pi$

b) $y = e^{-x}\sqrt{\sin x}, 0 \le x \le \pi$ $S = \frac{\pi}{5}(e^{-2\pi}+1)$

Ověřte, že délka křivky $y = \ln(1-x^2)$ v intervalu $[-\frac{1}{2}, \frac{1}{2}]$ je $L = -1 + 2\ln 3$