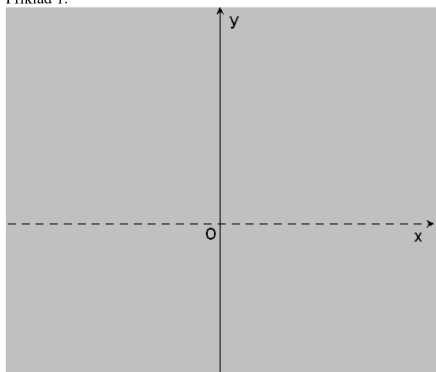


$$1. f(x, y) = \frac{x}{y}.$$

$$y \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \neq 0\}.$$

Příklad 1:



$$3. f(x, y) = \frac{x^2 + y}{x^2 - y^2}.$$

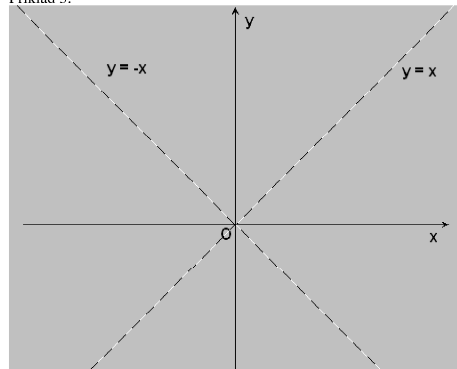
$$x^2 - y^2 \neq 0$$

$$(x - y) \cdot (x + y) \neq 0$$

$$x \neq y \wedge x \neq -y.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x \neq y \wedge x \neq -y\}.$$

Příklad 3:

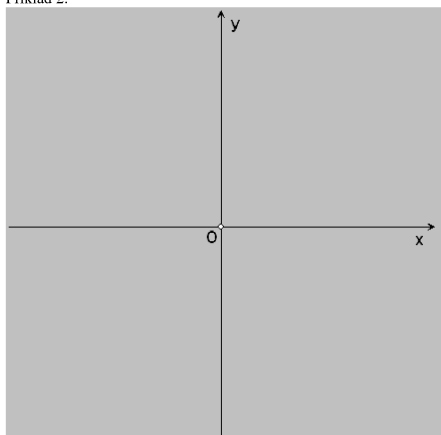


$$2. f(x, y) = \frac{3}{x^2 + y^2}.$$

$$x^2 + y^2 \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; (x, y) \neq (0, 0)\}.$$

Příklad 2:

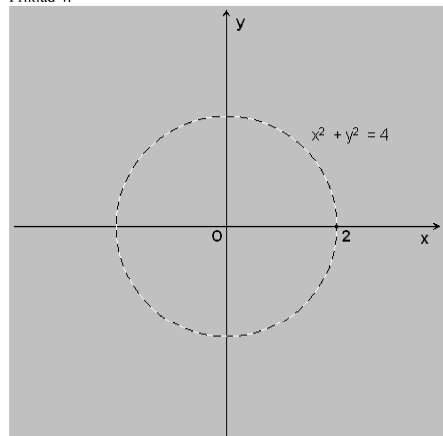


$$4. f(x, y) = \frac{xy}{4 - x^2 - y^2}.$$

$$4 - x^2 - y^2 \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \neq 4\}.$$

Příklad 4:

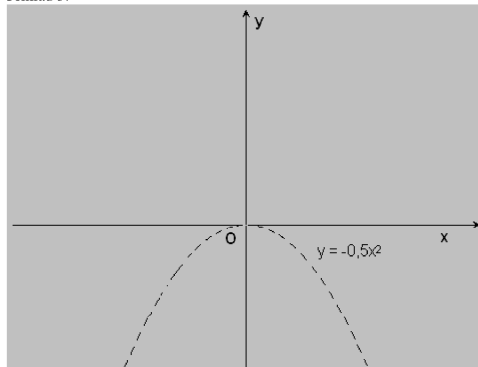


$$5. f(x, y) = \frac{x^2 - 1}{x^2 + 2y}.$$

$$x^2 + y \neq 0 \Rightarrow y \neq -\frac{1}{2}x^2.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \neq -\frac{1}{2}x^2\}.$$

Příklad 5:

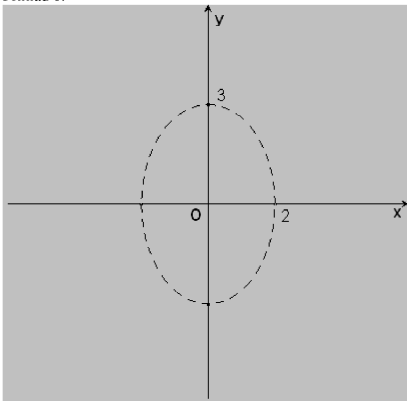


$$6. f(x, y) = \frac{x+y}{9x^2 + 4y^2 - 36}.$$

$$9x^2 + 4y^2 - 36 \neq 0 \Rightarrow 9x^2 + 4y^2 \neq 36.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; 9x^2 + 4y^2 \neq 36\}.$$

Příklad 6:

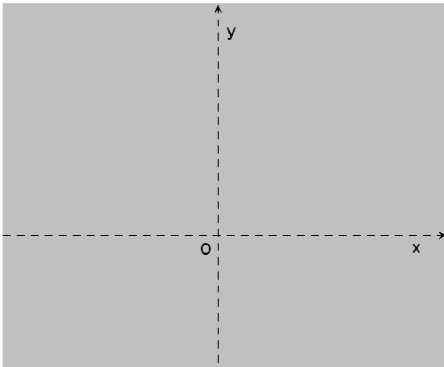


$$7. f(x, y) = \frac{1}{xy}.$$

$$xy \neq 0 \Rightarrow x \neq 0 \vee y \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x \neq 0 \vee y \neq 0\}.$$

Příklad 7:



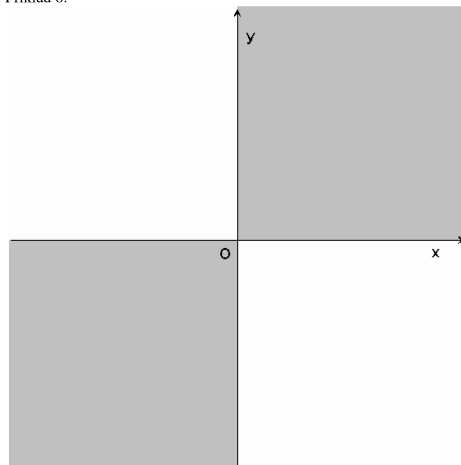
$$8. f(x, y) = \sqrt{xy}.$$

$$xy \geq 0$$

$$(x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0).$$

$$D_f = \{(x, y) \in \mathbb{R}^2; (x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0)\}.$$

Příklad 8:

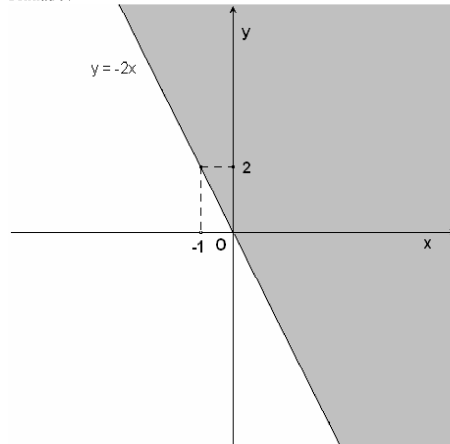


$$9. f(x, y) = \sqrt{2x + y}.$$

$$2x + y \geq 0 \Rightarrow y \geq -2x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \geq -2x\}.$$

Příklad 9:

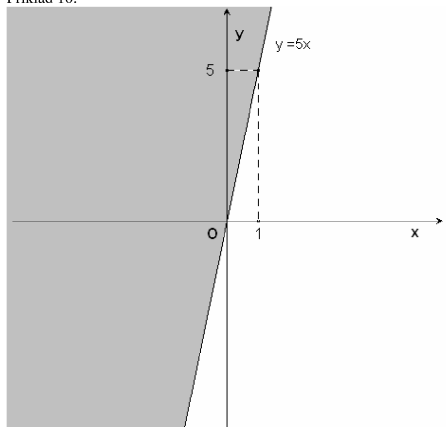


$$10. f(x, y) = \frac{x}{\sqrt{y-5x}}.$$

$$y - 5x \geq 0 \rightarrow y \geq 5x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \geq 5x\}.$$

Příklad 10:



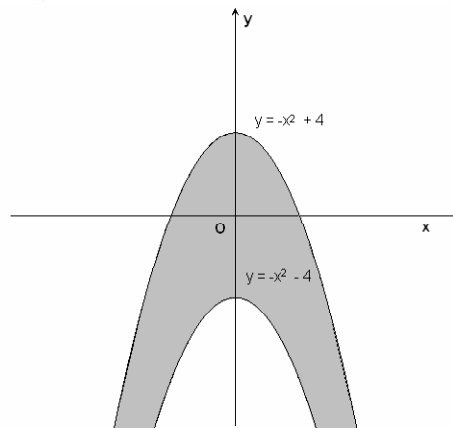
$$12. f(x, y) = \sqrt{16 - (x^2 + y)^2}.$$

$$16 - (x^2 + y)^2 \geq 0$$

$$\begin{aligned} \text{Odtud } (x^2 + y)^2 \leq 16 &\Rightarrow -4 \leq x^2 + y \leq 4 \\ 4 &\Rightarrow -4 - x^2 \leq y \leq 4 - x^2. \end{aligned}$$

$$D_f = \{(x, y) \in \mathbb{R}^2; -4 - x^2 \leq y \leq 4 - x^2\}.$$

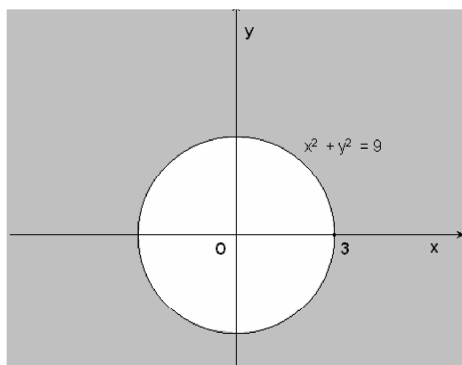
Příklad 12:



$$11. f(x, y) = \sqrt{x^2 + y^2 - 9}.$$

$$x^2 + y^2 - 9 \geq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \geq 9\}.$$



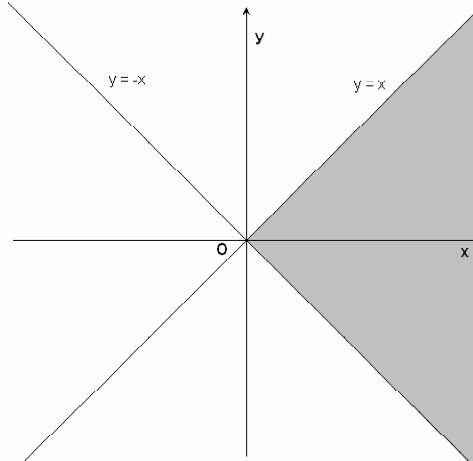
$$13. f(x) = 2\sqrt{x+y} - 5\sqrt{x-y}.$$

$$x + y \geq 0 \wedge x - y \geq 0$$

$$y \geq -x \wedge y \leq x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; -x \leq y \leq x\}.$$

Příklad 13:



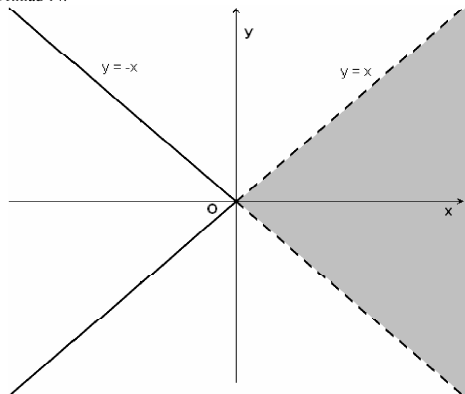
$$14. f(x, y) = \frac{1}{\sqrt{x+y}} + \frac{1}{\sqrt{x-y}}.$$

$$x + y > 0 \wedge x - y > 0$$

$$y > -x \wedge y < x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; -x < y < x\}.$$

Příklad 14:



$$15. f(x, y) = \sqrt{4-x^2} + \sqrt{1-y^2}.$$

$$4-x^2 \geq 0 \quad \wedge \quad 1-y^2 \geq 0$$

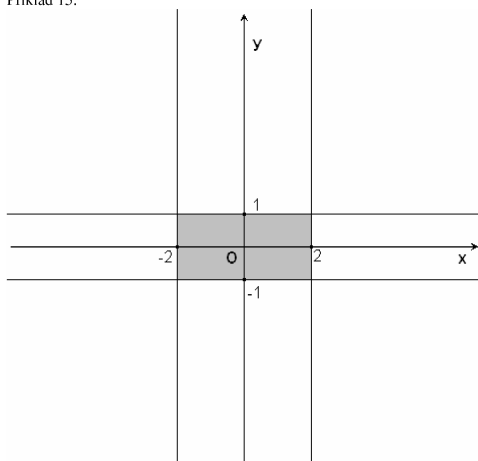
$$x^2 \leq 4 \quad \wedge \quad y^2 \leq 1$$

$$|x| \leq 2 \quad \wedge \quad |y| \leq 1$$

$$-2 \leq x \leq 2 \quad \wedge \quad -1 \leq y \leq 1$$

$$D_f = \{(x, y) \in \mathbb{R}^2; -2 \leq x \leq 2 \wedge -1 \leq y \leq 1\}.$$

Příklad 15:

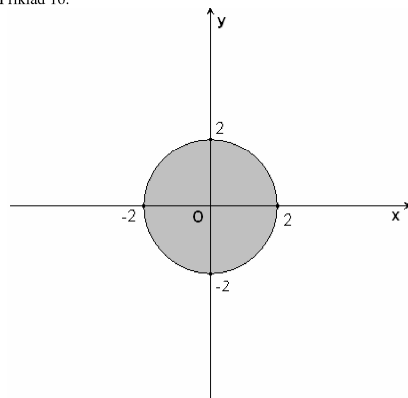


$$16. f(x, y) = (x^2 + y^2) \sqrt{4-x^2-y^2}.$$

$$4-x^2-y^2 \geq 0 \quad \Rightarrow \quad x^2 + y^2 \leq 4.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4\}.$$

Příklad 16:

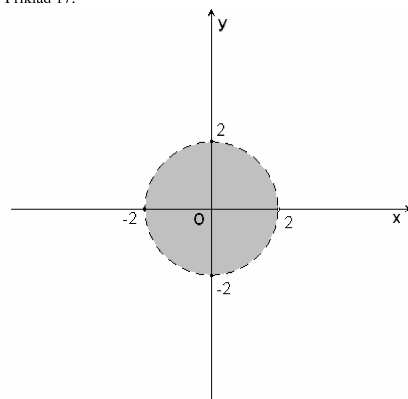


$$17. f(x, y) = \frac{x^2+y^2}{\sqrt{4-x^2-y^2}}.$$

$$4-x^2-y^2 > 0 \quad \Rightarrow \quad x^2 + y^2 < 4.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 < 4\}.$$

Příklad 17:

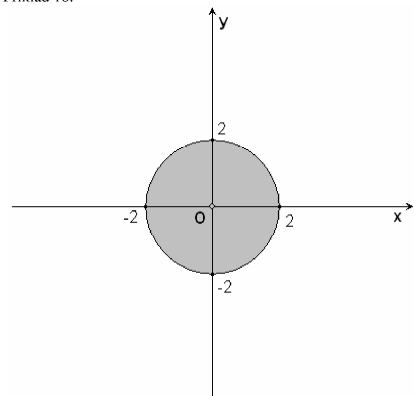


$$18. f(x, y) = \frac{\sqrt{4-x^2-y^2}}{x^2+y^2}.$$

$$4-x^2-y^2 \geq 0 \quad \wedge \quad x^2 + y^2 \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4 \wedge (x, y) \neq (0, 0)\}.$$

Příklad 18:



$$19. f(x, y) = \sqrt{(x^2 + y^2 - 25)(16 - x^2 - y^2)}.$$

$$(x^2 + y^2 - 25) \cdot (16 - x^2 - y^2) \geq 0.$$

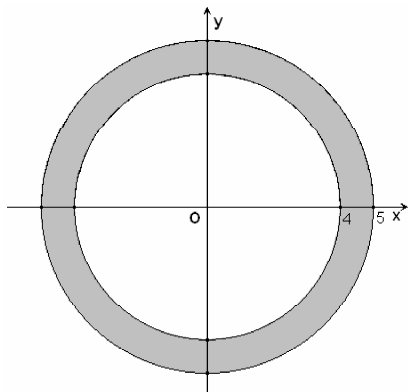
$$(x^2 + y^2 - 25 \geq 0 \wedge 16 - x^2 - y^2 \geq 0) \quad \vee \quad (x^2 + y^2 - 25 \leq 0 \wedge 16 - x^2 - y^2 \leq 0)$$

$$(x^2 + y^2 \geq 25 \wedge x^2 + y^2 \leq 16) \quad \vee \quad (x^2 + y^2 \leq 25 \wedge x^2 + y^2 \geq 16)$$

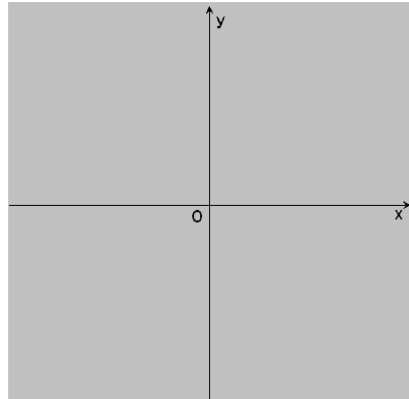
$$(x, y) \in \emptyset \quad \vee \quad 16 \leq x^2 + y^2 \leq 25.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; 16 \leq x^2 + y^2 \leq 25\}.$$

Příklad 19:



Příklad 20:



$$20. f(x, y) = \sqrt{\frac{x^2 - 2x + y^2}{4x - x^2 - y^2}}.$$

$$\frac{x^2 - 2x + y^2}{4x - x^2 - y^2} \geq 0.$$

$$(x^2 - 2x + y^2 \geq 0 \wedge 4x - x^2 - y^2 > 0) \quad \vee \quad (x^2 - 2x + y^2 \leq 0 \wedge 4x - x^2 - y^2 < 0).$$

Protože

$$x^2 - 2x + y^2 = x^2 - 2x + 1 - 1 + y^2 = (x - 1)^2 + y^2 - 1$$

$$\begin{aligned} 4x - x^2 - y^2 &= -(x^2 - 4x + y^2) = -(x^2 - 4x + 4 - 4 + y^2) = -[(x - 2)^2 + y^2 - 4] \\ &= -(x - 2)^2 - y^2 + 4, \end{aligned}$$

máme

$$\left[ (x - 1)^2 + y^2 - 1 \geq 0 \wedge -(x - 2)^2 - y^2 + 4 > 0 \right] \vee \left[ (x - 1)^2 + y^2 - 1 \leq 0 \wedge -(x - 2)^2 - y^2 + 4 < 0 \right]$$

$$\left[ (x - 1)^2 + y^2 \geq 1 \wedge (x - 2)^2 + y^2 < 4 \right] \quad \vee \quad \left[ (x - 1)^2 + y^2 \leq 1 \wedge (x - 2)^2 + y^2 > 4 \right]$$

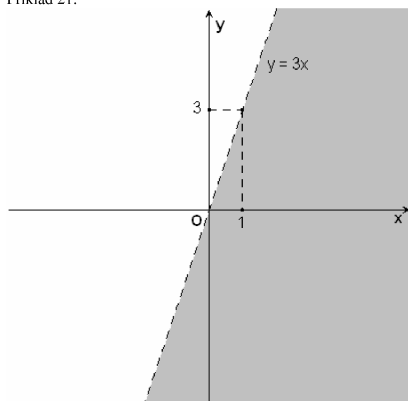
$$D_f = \{(x, y) \in \mathbb{R}^2; \}. \}$$

$$21. f(x, y) = \ln(3x - y).$$

$$3x - y > 0 \quad \rightarrow \quad y < 3x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y < 3x\}.$$

Příklad 21:

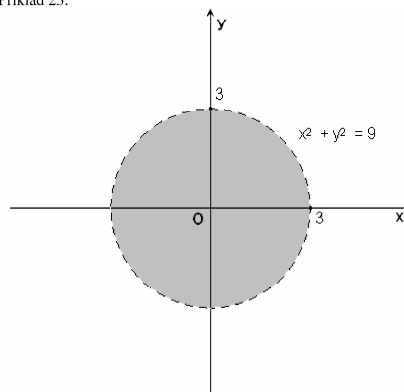


$$23. f(x, y) = \ln(9 - x^2 - y^2).$$

$$9 - x^2 - y^2 > 0 \quad \rightarrow \quad x^2 + y^2 < 9.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 < 9\}.$$

Příklad 23:

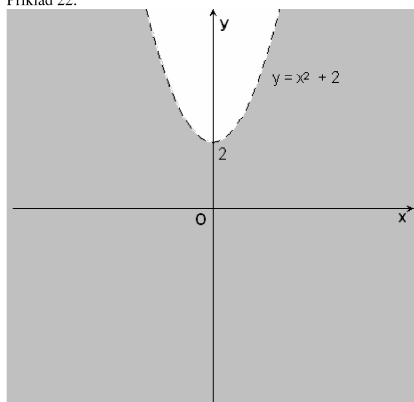


$$22. f(x, y) = \log(x^2 - y + 2).$$

$$x^2 - y + 2 > 0 \quad \Rightarrow \quad y < x^2 + 2.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y < x^2 + 2\}.$$

Příklad 22:



$$24. f(x, y) = x - \ln\left(y + \frac{1}{x}\right).$$

$$y + \frac{1}{x} > 0 \quad \wedge \quad x \neq 0$$

$$\frac{xy + 1}{x} > 0 \quad \wedge \quad x \neq 0$$

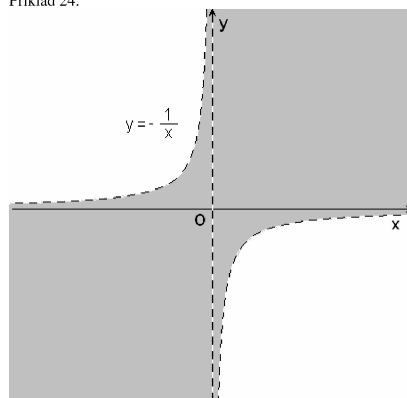
$$[(xy + 1 > 0 \wedge x > 0) \vee (xy + 1 < 0 \wedge x < 0)]$$

$$\wedge \quad x \neq 0$$

$$\left[ \left( y > -\frac{1}{x} \wedge x > 0 \right) \vee \left( y < -\frac{1}{x} \wedge x < 0 \right) \right]$$

$$\wedge \quad x \neq 0$$

Příklad 24:



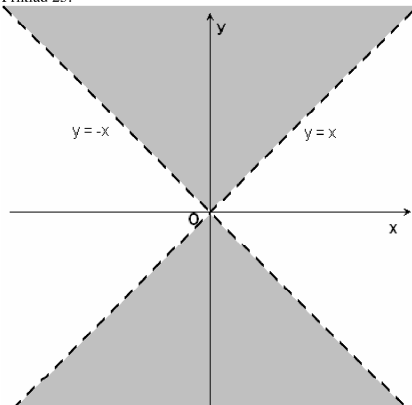
25.  $f(x, y) = \ln \frac{x+y}{x-y}$ .

$$\frac{x+y}{x-y} > 0$$

$$(x+y > 0 \wedge x-y > 0) \vee (x+y < 0 \wedge x-y < 0)$$

$$(y > -x \wedge y < x) \vee (y < -x \wedge y > x)$$

Příklad 25:



27.  $f(x, y) = \frac{x+y}{\ln(1-x^2-y^2)}$ .

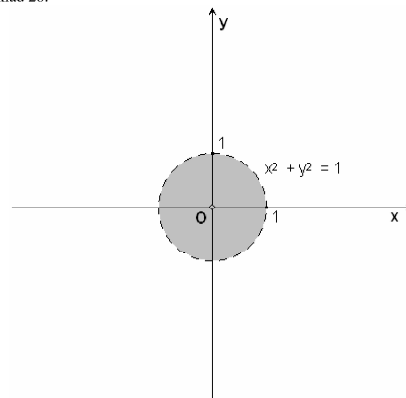
$$\ln(1-x^2-y^2) \neq 0 \quad \wedge \quad 1-x^2-y^2 > 0$$

$$1-x^2-y^2 \neq 1 \quad \wedge \quad x^2+y^2 < 1$$

$$x^2+y^2 \neq 0 \quad \wedge \quad x^2+y^2 < 1.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x^2+y^2 < 1 \wedge (x, y) \neq (0, 0)\}.$$

Příklad 28:



26.  $f(x, y) = \ln(x - \ln y)$ .

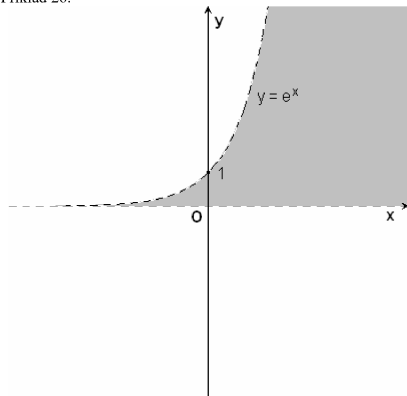
$$x - \ln y > 0 \quad \wedge \quad y > 0$$

$$x - \ln y > 0 \Rightarrow \ln y < x \Rightarrow \ln y < \ln e^x$$

$$\ln e^x \Rightarrow y < e^x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y < e^x \wedge y > 0\}.$$

Příklad 26:



28.  $f(x, y) = \frac{\sqrt{x-y^2}}{\ln(1-x^2-y^2)}$ .

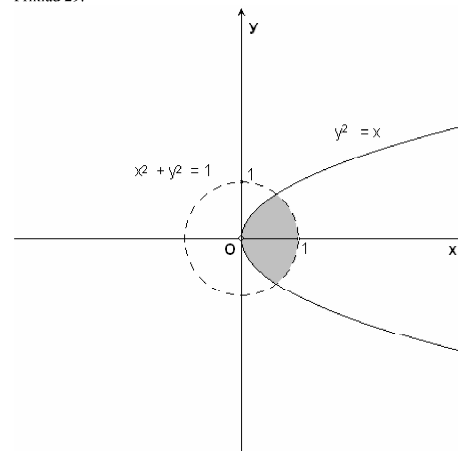
$$x-y^2 \geq 0 \quad \wedge \quad \ln(1-x^2-y^2) \neq 0 \quad \wedge \quad 1-x^2-y^2 > 0$$

Odtud

$$x \geq y^2 \quad \wedge \quad x^2+y^2 \neq 0 \quad \wedge \quad x^2+y^2 < 1.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x \geq y^2 \wedge x^2+y^2 < 1 \wedge (x, y) \neq (0, 0)\}.$$

Příklad 29:



29.  $f(x, y) = \ln[y \cdot \ln(x - y)]$ .

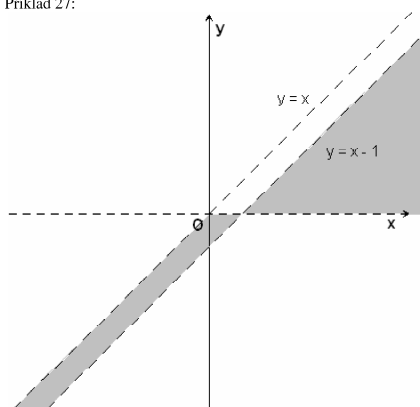
$$y \cdot \ln(x - y) > 0 \quad \wedge \quad x - y > 0$$

1)

$$\begin{aligned} y \cdot \ln(x - y) > 0 &\Rightarrow [y > 0 \wedge \ln(x - y) > 0] \quad \vee \quad [y < 0 \wedge \ln(x - y) < 0] \\ &\Rightarrow (y > 0 \wedge x - y > 1) \quad \vee \quad (y < 0 \wedge x - y < 1) \\ &\Rightarrow (y > 0 \wedge y < x - 1) \quad \vee \quad (y < 0 \wedge y > x - 1). \end{aligned}$$

2)  $x - y > 0 \Rightarrow y < x$ .

Příklad 27:

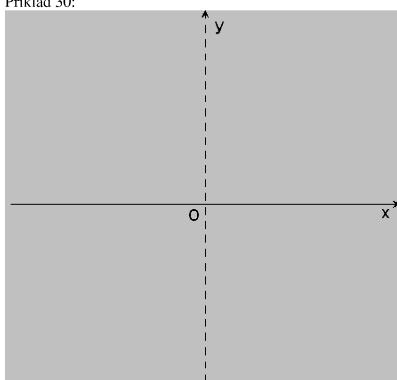


30.  $f(x, y) = \sin \frac{y}{x}$ .

$$x \neq 0.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; x \neq 0\}.$$

Příklad 30:

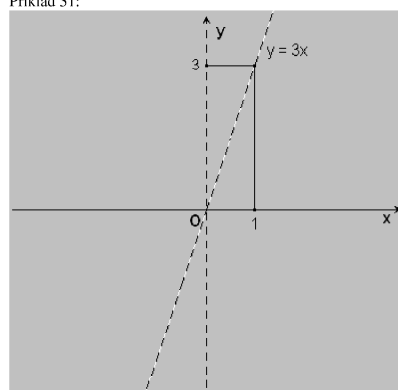


31.  $f(x, y) = \cos \frac{1}{3x - y}$ .

$$3x - y \neq 0 \Rightarrow y \neq 3x.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \neq 3x\}.$$

Příklad 31:





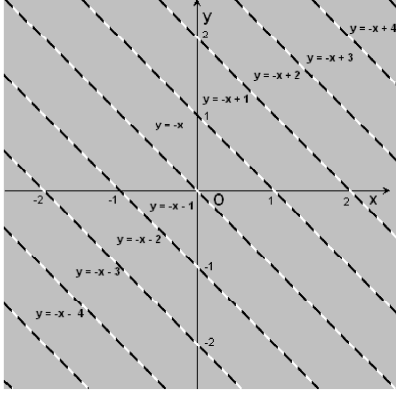
32.  $f(x, y) = \cot \pi (x + y).$

$$\pi (x + y) \neq k\pi, k \in \mathbb{Z}$$

$$\Rightarrow x + y \neq k \Rightarrow y \neq -x + k.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \neq -x + k, k \in \mathbb{Z}\}.$$

Příklad 32:



34.  $f(x, y) = \sqrt{y \cdot \cos x}.$

$$y \cdot \cos x \geq 0$$

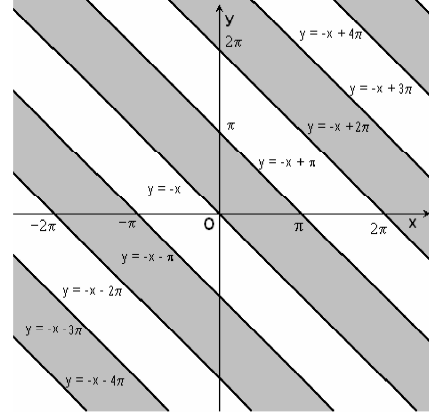
$$(y \geq 0 \wedge \cos x \geq 0) \quad \vee \quad (y \leq 0 \wedge \cos x \leq 0)$$

$$y \geq 0 \quad \wedge \quad -\frac{\pi}{2} + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$\vee$$

$$y \leq 0 \quad \wedge \quad \frac{\pi}{2} + 2k\pi \leq x \leq \frac{3}{2}\pi + 2k\pi, k \in \mathbb{Z}.$$

Příklad 34:



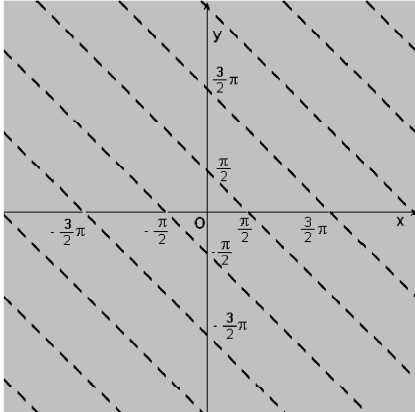
33.  $f(x, y) = \sqrt{\sin (x + y)}.$

$$\sin (x + y) \geq 0 \Rightarrow 2k\pi \leq x + y \leq (2k + 1)\pi, k \in \mathbb{Z}$$

$$\Rightarrow -x + 2k\pi \leq y \leq -x + (2k + 1)\pi$$

$$D_f = \{(x, y) \in \mathbb{R}^2; -x + 2k\pi \leq y \leq -x + (2k + 1)\pi, \\ k \in \mathbb{Z}\}.$$

Příklad 33:

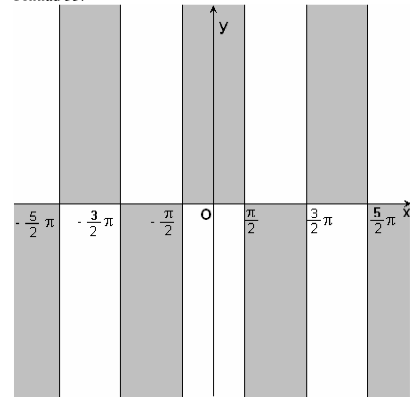


35.  $f(x, y) = \sqrt{\sin (x^2 + y^2)}.$

$$\sin (x^2 + y^2) \geq 0 \Rightarrow 0 + 2k\pi \leq x^2 + y^2 \leq \pi + 2k\pi, k \in \mathbb{Z}^+.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; 2k\pi \leq x^2 + y^2 \leq (2k + 1)\pi, k \in \mathbb{Z}^+\}.$$

Příklad 35:



36.  $f(x, y) = \ln \cos(x + y)$ .

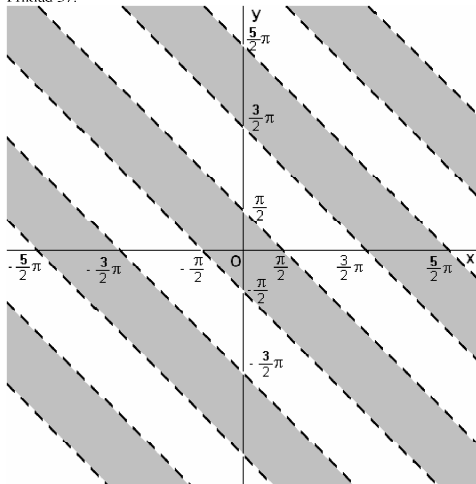
$$\cos(x + y) > 0 \Rightarrow -\frac{\pi}{2} + 2k\pi < x + y < \frac{\pi}{2} + 2k\pi$$

$$\Rightarrow -x - \frac{\pi}{2} + 2k\pi < y < -x + \frac{\pi}{2} + 2k\pi$$

$$D_f = \{(x, y) \in \mathbb{R}^2;$$

$$-x + \frac{\pi}{2}(4k - 1) < y < -x + \frac{\pi}{2}(4k + 1), k \in \mathbb{Z}\}.$$

Příklad 37:



38.  $f(x, y) = \arcsin(1 - x) + \arccos \frac{y}{x^2}$ .

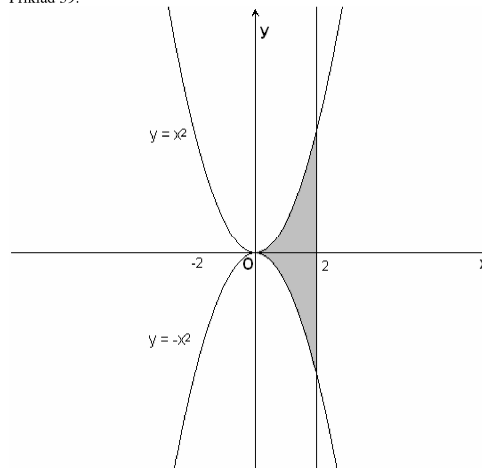
$$-1 \leq 1 - x \leq 1 \quad \wedge \quad -1 \leq \frac{y}{x^2} \leq 1$$

$$-2 \leq -x \leq 0 \quad \wedge \quad -x^2 \leq y \leq x^2$$

$$2 \geq x \geq 0 \quad \wedge \quad -x^2 \leq y \leq x^2.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; 0 \leq x \leq 2 \wedge -x^2 \leq y \leq x^2\}.$$

Příklad 39:



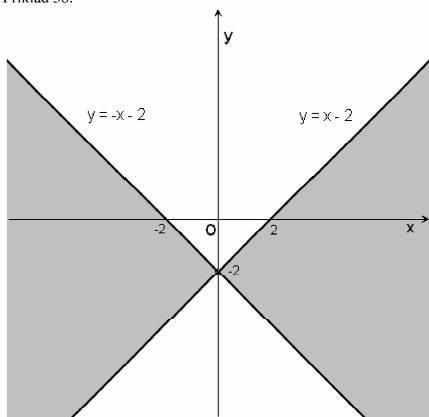
37.  $f(x, y) = \arccos \frac{y+2}{x}$ .

$$-1 \leq \frac{y+2}{x} \leq 1.$$

pro  $x > 0$  :  $-x \leq y + 2 \leq x \Rightarrow -x - 2 \leq y \leq x - 2$ ,

pro  $x < 0$  :  $-x \geq y + 2 \geq x \Rightarrow -x - 2 \geq y \geq x - 2$ .

Příklad 38:



39.  $f(x, y) = \arctan \frac{x}{x^2 + y}$ .

$$x^2 + y \neq 0 \Rightarrow y \neq -x^2.$$

$$D_f = \{(x, y) \in \mathbb{R}^2; y \neq -x^2\}.$$

Příklad 40:

